

Exercise 30

For the following exercises, use synthetic division to find the quotient.

$$(x^4 - 3x^2 + 1) \div (x - 1)$$

Solution

Solving $x - 1 = 0$ gives $x = 1$; this is the number that goes on the top left. Write out all the coefficients of the dividend to the right.

$$\begin{array}{r|rrrrr} 1 & 1 & 0 & -3 & 0 & 1 \\ \hline & & & & & \end{array}$$

Bring down the leading coefficient.

$$\begin{array}{r|rrrrr} 1 & 1 & 0 & -3 & 0 & 1 \\ \hline & 1 & & & & \end{array}$$

Multiply the top left number by the number brought down and put the result under the second coefficient of the dividend.

$$\begin{array}{r|rrrrr} 1 & 1 & 0 & -3 & 0 & 1 \\ \hline & 1 & 1 & & & \end{array}$$

Add the numbers in the second column.

$$\begin{array}{c|cccc} 1 & 1 & 0 & -3 & 0 & 1 \\ \hline & & 1 & & & \\ \hline & 1 & 1 & & & \end{array}$$

Multiply this sum of the second column by the top left number and put it in the next column.

$$\begin{array}{c|cccc} 1 & 1 & 0 & -3 & 0 & 1 \\ \hline & & 1 & 1 & & \\ \hline & 1 & 1 & & & \end{array}$$

Add the numbers in the third column.

$$\begin{array}{c|cccc} 1 & 1 & 0 & -3 & 0 & 1 \\ \hline & & 1 & 1 & & \\ \hline & 1 & 1 & -2 & & \end{array}$$

Multiply this sum of the third column by the top left number and put it in the next column.

$$\begin{array}{c|cccc} 1 & 1 & 0 & -3 & 0 & 1 \\ \hline & & 1 & 1 & -2 & \\ \hline & 1 & 1 & -2 & & \end{array}$$

Add the numbers in the fourth column.

$$\begin{array}{c|cccc} 1 & 1 & 0 & -3 & 0 & 1 \\ \hline & & 1 & 1 & -2 & \\ \hline & 1 & 1 & -2 & -2 & \end{array}$$

Multiply this sum of the fourth column by the top left number and put it in the next column.

$$\begin{array}{c|cccc} 1 & 1 & 0 & -3 & 0 & 1 \\ \hline & & 1 & 1 & -2 & -2 \\ \hline & 1 & 1 & -2 & -2 & \end{array}$$

Add the numbers in the fifth column.

$$\begin{array}{r|rrrrr}
 1 & 1 & 0 & -3 & 0 & 1 \\
 & & & & & \\
 & & 1 & 1 & -2 & -2 \\
 \hline
 & 1 & 1 & -2 & -2 & -1
 \end{array}$$

This final result is the remainder, and the numbers to the left are the coefficients of the quotient, which is $x^3 + x^2 - 2x - 2$.

$$(x^4 - 3x^2 + 1) \div (x - 1) = x^3 + x^2 - 2x - 2 + \frac{-1}{x - 1}$$